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Fixed Point Results in Complete Random Convex Metric Spaces

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ABSTRACT: In this paper, using some conditions of (sub) compatibility between a set-valued mapping and a single- valued mapping, we establish a necessary and sufficient condition for set-valued generalized nonexpansive mappings to have a unique common fixed point in complete random convex metric spaces. The results improve, extend and develop the main results in [2-7] and [18] for random variable.

Key words convex metric spaces, (sub) compatible mapping, set-valued generalized nonexpansive mapping, random fixed points

I. INTRODUCTION AND PRELIMINARIES

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of random operators forms a central topic in this area. The Prague school of probabilistic initiated its study in the 1950. In recent years, the study of random fixed points has attracted much attention. In particular, random iteration schemes leading to random fixed point of random operators have been discussed in [19-22].

Sessa introduced the concept of weakly commuting mappings in[1]. Two single-valued mappings T and I of a metric space (X, d) into itself are said to be weakly commuting if

$$d(TIx, ITx) \leq d(Tx, Ix)$$

for all x in X. Recently, Fisher and Sessa [2] proved the following generalization of a theorem of Gregus [3] for two weakly commuting mappings T and I.

Theorem 2.1 Let C be a nonempty closed convex subset of a Banach space X, T and I be two weakly commuting mappings of C into itself satisfying the inequality

$\|Tx - Ty\| \leq a\|Ix - Iy\| + (1 - a)\max\{\|Tx - Ix\|, \|Ty - Iy\|\}$

for all x, y in C, where $0 < a < 1$. If I is linear, nonexpansive in C and such that IC contains TC, then T and I have a unique common fixed point in C.

Mukherjee and Verma [4] proved that Theorem 1.1 remains true when "I is linear" is replaced by "I is affine". In a recent paper jungck [5] showed that Theorem 1.1 can be generalized by substituting compatibility for weak commutativity and continuity for the nonexpansive requirement.

The purpose of this paper is to further extend Theorem 1.1, with the help of some conditions of compatibility between a set-valued mapping and a single-valued mapping. We will establish a necessary and sufficient condition and a sufficient condition for set-valued generalized nonexpansive mappings to have a unique common fixed point in complete convex metric spaces. Our results are motivated by main results of Fisher and Sessa [2], Mukherjee and Verma [4], Jungck [5], Li [6] , Fisher [7], Gregus [3] Choudhary [22] and the others

II. BASIC DEFINITIONS AND LEMMAS

Let (X, d) be a complete metric space and $B(X)$ be the set of all nonempty bounded subsets of X. As in $[8 - 11]$, let $\delta(A, B)$ be the function defined by
$\delta(A, B) = \sup\{d(a, b): a \in A, b \in B\}$,
For all A, B in $B(X)$. If $A = \{a\}$ is singleton, we write $\delta(A, B) = \delta(a, B)$ and if B also consists of a single point b we write $\delta(A, B) = d(a, b)$. It follows immediately from the definition that
$\delta(A, B) = \delta(B, A) \geq 0, \quad \delta(A, B) \leq \delta(A, C) + \delta(C, B)$,
$\delta(A, A) = dianA, \delta(A, B) = 0 \Leftrightarrow A = B = \{a\}$
for all A, B, C in $B(X)$.

Definition 2.1 ([8, 10]) A sequence $\{A_n\}$ of subsets of X is said to converge to a subset A of X if

(1) Given $a \in A$, there is a sequence $\{a_n\}$ in X such that $a_n \in A_n$ for $n = 1,2,....$ and $\{a_n\}$ converges to a;

(2) Given $\varepsilon > 0$, there exists a positive integer N such that $A_n \subseteq A_\varepsilon$, for $n > N$ where A_ε, is the union of all open spheres with centers in A and radius ε

Lemma 2.1 [8, 10] If $\{A_n\}$ and $\{B_n\}$ are sequence in $B(X)$ converging respectively to A and B in $B(X)$, then sequence $\{\delta(A_n, B_n)\}$ converges to $\delta(A, B)$.

Lemma 2.2 ([10]) Let A_n be a sequence in $B(X)$ and y be a point of X such that $\delta(A_n, y) \to 0$, then the sequence $\{A_n\}$ converges to the set $\{y\}$ in $B(X)$.

Definition 2.2 ([8,10]) A set-valued mapping F of X into $B(X)$ is said to be continuous at $x \in X$ if the sequence $\{Fx_n\}$ in $B(X)$ converges to Fx whenever $\{x_n\}$ is a sequence in X converging to x in X, F is said to be continuous on X if it is continuous at every point of X.

In a recent paper, Jungck[12] made an extension of weak commutivity in the following way:

Definition 2.3 Two single-valued self-mappings f and g of metric space (X, d) are compatible if $d(fgx_n, gfx_n) \to 0$. whenever $\{x_n\}$ is a sequence in X such that $fx_n \to p, gx_n \to p$ for some point in X.

It can be seen that two weakly commuting mappings are compatible but the converse is false.

Definition 2.4 The mappings $f: X \to X$ and $F: X \to B(X)$ are compatible if $\delta(Ffx_n, fFx_n) \to 0$, whenever $\{x_n\}$ is a sequence in X such that $fFx_n \in B(X), Fx_n \to \{p\}$ and $fx_n \to p$ for some point p in X.

Definition 2.5 The mappings $f: X \to X$ and $F: X \to B(X)$ are sub compatible if $\{p \in X: Fp = \{fp\}\} \subseteq \{p \in X: Ffp = fFp\}$.

Definition 2.6 ([14, 15]) A mapping $W: X \times X \times [0,1] \to X$ is called a convex structure on X, if for any $(x,\ y,\ \lambda) \in X \times X \times [0,1]$ and any $u \in X$

$$d\big(u, W(x, y, \lambda)\big) \leq \lambda d(u, x) + (1 - \lambda) d(u, y).$$

A metric space with a convex structure is called a convex metric space. A nonempty subset K of X is said to be convex if $W(x, y, \lambda) \in K$ for all x, y in K and for all $\lambda \in [0,1]$.

Let ϕ be the set of all the functions $\varphi: [0, \infty) \to [0, \infty)$ which is continuous from the right and $\varphi(2p) < p$ for all $p > 0$.

Lemma 2.3[18] Let K be a nonempty closed subset of a complete metric space (X, d). If mapping $\mathrm{f: K \to K}$ and $\mathrm{F: K \to B(K)}$ satisfy the following inequality

$$\delta(Fx, Fy) \leq ad(fx, fy) + \varphi(2max\{\delta(Fx, fx), \delta(Fy, fy)\}) \quad (2.1)$$

For all x, y in K, where $0 \leq a < 1$ and $\varphi \in \phi$, then

(1) F and f have at most one common fixed point u in K, and further $F\big(t, u(t)\big) = \{u(t)\}$,

(2) If $\{x_n\}$ is a sequence in K such that $\delta(Fx_n, fx_n) \to 0$, then there exists a $u \in K$ such that $Fx_n \to \{u\}$ and $fx_n \to u$.

Throughout this paper, (Ω, Σ) denotes a measurable space, C is non empty subset of K

Definition 2.7 Measurable function: A function $f : \Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \Sigma$ foe every Borel subset B of X.

Definition 2.8 Random operator: A function $f : \Omega \times C \to C$ is said to be random operator, if $F(., X) : \Omega \to C$ is measurable for every X $\in$ C

Definition 2.9Continuous Random operator: A random operator $f : \Omega \times C \to c$ is said to be continuous if for fixed $t \in \Omega, f(t,.) : C \to C\ is\ continuous$

Definition 2.10. **Random fixed point:** A measurable function $g : \Omega \to C$ is said to be random fixed point of the random operator $f : \Omega \times C \to C, if\ f\big(t, g(t)\big) = g(t), \forall\, t \in \Omega$

Definition 2.11: Let (X, d) be a metric space and (Ω, Σ) is a measurable space, J= [0,1]. A mapping $W: X \times X \times J \to X$,is called a convex structure on X for random operator if for each $(x(t), y(t), \delta) \in X \times X \times J$ and u(t) $\in X$

$d(u(t), W(t, (x(t), y(t), \delta)) \leq \delta\, d\big(u(t), x(t)\big) + (1 - \delta) d(u(t), y(t))$

A metric space X together with a convex structure w and random operator is called a convex random metric space.

Definition 2.12: A nonempty subset K of a convex random metric space (X,d) with a convex structure w is said to be convex if for all (x(t), y(t), δ) $\in$ K x Kx J, w [t, (x(t), y(t), δ)] $\in$ K

Throughout this paper, a random convex metric space will be denoted by (X, d, W).It is easy to know that, any linear normed space and their convex subsets are convex metric spaces. It is to be noted that the definitions 2.1 to 2.6 and lemma 2.1 to 2.3 all are true for random operator

III. MAIN RESULTS

Theorem 3.1 Let K be a nonempty closed subset of a complete metric space (X,d). (Ω,Σ) denotes a measurable space; C is non empty subset of K, let F be a mapping of K into $B(K)$ and f a mapping of K into itself satisfying the inequality (2.1). if F and f satisfy one of the following conditions:

(1) (F,f) are compatible and f is continuous:
(2) (F,f) are compatible $FK \subseteq fK$ and F is continuous:
(3) (F,f) are subcompatible and f is surjective,

Then F and f have a unique common fixed point $u(t)$ in K such that $F(t,u(t)) = \{u(t)\}$ if $\inf\left\{\delta\left(F(t,x(t)), f(t,x(t))\right) : x(t) \in K\right\} = 0$.

Proof Suppose that $u(t)$ is a unique common fixed point of F and f in K, i.e. $u(t) = f(t,u(t)) \in F(t,u(t))$. Using the inequality (2.1), we obtain that

$$\delta\left(F(t,u(t)),u(t)\right) \le \delta\left(F(t,u(t)),F(t,u(t))\right) \le \varphi\left(2\delta\left(F(t,u(t)),u(t)\right)\right),$$

Which implies $F(t,u(t)) = \{u(t)\}$ by $\varphi \in \phi$, and so $\inf\left\{\delta\left(F(t,x(t)), f(t,x(t))\right) : x(t) \in K\right\} = 0$. therefore, necessity is proved. To prove sufficient let $\{x_n(t)\}$ is a sequence such that

$$\delta\left(F(t,x_n(t)), f(t,x_n(t))\right) \to \inf\left\{\delta\left(F(t,x(t)), f(t,x(t))\right) : x(t) \in K\right\} = 0.$$

It follows from Lemma 2.3 (2) that the sequence $\{f(t,x_{n(t)})\}$ converges to some point $u(t) \in K$ and the sequence of sets $\{F(t,x_n(t))\}$ converges to the set $\{u(t)\}$.

Now suppose that (1) holds. Then the sequence $\{f^2(t,x_n(t))\}$ and $\{fF(t,x_n(t))\}$ converge to $f(t,u(t))$ and $\{f(t,u(t))\}$ respectively. Since

$$\delta\left(Ff(t,x_n(t)), f(t,u(t))\right) \le \delta\left(Ff(t,x_n(t))\right), fF(t,x_n(t))$$
$$+\delta\left(fF(t,x_n(t)), f(t,u(t))\right)$$

And (F,f) are compatible, we have that $\delta\left(Ff(t,x_n(t)), f(t,u(t))\right) \to 0$, which implies the sequence $\{fF(t,x_n(t))\}$ converges to $\{f(t,u(t))\}$ by Lemma 2.2. Using the inequality (2.1) we have that

$$\delta\left(fF(t,x_n(t)), F(t,x_n(t))\right) \le ad\left(f^2(t,x_n(t)), f(t,x_n(t))\right)$$
$$+\varphi(2\max\left\{\delta\left(Ff(t,x_n(t)), f^2(t,x_n(t))\right), \delta(F(t,x(t)), f(t,x(t))\right\}$$

Which implies, as $n \to \infty$, that $d\left(f(t,u(t)),u(t)\right) \le ad\left(f(t,u(t)),u(t)\right)$ by Lemma 2.1 and hence $f(t,u(t)) = u(t)$ Using again the inequality (2.1), we have

$$\delta\left(F(t,x_n(t)), F(t,u(t))\right) \le ad\left(f(t,x_n(t)), f(t,u(t)\right)$$
$$+\varphi(2\max\begin{Bmatrix}\delta\left(F(t,x_n(t)), f(t,x_n(t)), x_n(t)\right), \\ \delta\left(F(t,u(t)), f(t,u(t))\right)\end{Bmatrix}$$

and this implies, as $n \to \infty$, that

$$\delta\left(u(t), F(t,u(t))\right) \le \varphi\left(2\delta\left(F(t,u(t)),u(t)\right)\right)$$

Then $F(t,u(t)) = \{u(t)\}$ and hence $u(t)$ is also a fixed point of F. therefore, by Lemma 2.3 (1) we know that $u(t)$ is the unique common fixed point F and f.

Now suppose that (2) holds. Then the sequence $\{Ff(t,x_n(t))\}$ converges to $F(t,u(t))$. Let $u_n(t)$ be an arbitrary point in $F(t,x_n(t))$ for $n = 1,2,\dots\dots$. Then since $d(u_n(t),u(t)) \le \delta\left(F(t,x_n(t)),u(t)\right)$ and F is continuous, we get that the sequence $\{F(t,u_n(t))\}$ converges to $F(t,u(t))$. Using inequality (2.1) we have that

$\delta\left(F(t,u_n(t)),F(t,x(t))\right)\le$
$ad\left(f(t,u_n(t)),f(t,x_n(t))\right)+\varphi(2\max\left\{\delta\left(F(t,u_n(t)),f(t,u_n(t))\right),\delta\left(F\left(t,x_n(t),f(t,x_n(t))\right)\right)\right\}\le$
$a\left[\delta\left(fF(t,x_n(t)),Ff(t,x_n(t))\right)+\delta\left(Ff(t,x_n(t)),f(t,x_n(t))\right)\right]$
$+\varphi\left(2\max\ \left\{\delta\left(F(t,u_n(t)),Ff(t,x_n(t))\right)+\delta\left(Ff(t,x_n(t)),fF(t,x_n(t))\right),\delta\left(F(t,x_n(t)),f(t,x_n(t))\right)\right\}\right)$
Since φ is right continuous and (F,f) are compatible, as $n\to\infty$, using (2.2) and Lemma 2.1, we obtain that
$$\delta\left(F(t,u(t)),u(t)\right)\le a\delta\left(F(t,u(t)),u(t)\right)+\varphi(2\delta\left(F(t,u(t)),u(t)\right). \quad (3.1)$$
But again using inequality (2.1), we deduce that
$\delta\left(F(t,u_n(t)),F(t,u_n(t))\right)\le ad\left(f(t,u_n(t)),f(t,u_n(t))\right)+\varphi(2\delta\left(F(t,u_n(t)),f(t,u_n(t))\right)$
$\le\varphi\left(2\delta\left(F(t,u_n(t)),Ff(t,x_n(t)\right)+2\delta(Ff(t,x_n(t)),fF(t,x_n(t))\right),$
Which implies, as $n\to\infty$, by the compatibility of (F,f) and Lemma 2.1 that
$\delta\left(F(t,u(t)),F(t,u(t))\right)\le\varphi\left(2\delta\left(F(t,u(t)),u(t)\right)\right),$
and hence $\delta\left(F(t,u(t)),F(t,u(t))\right)=0$. Form (3.1), it follows that $F(t,u(t))=\{u(t)\}$. Since $FK\subseteq fK$, there exists a point w in K such that $fw=u(t)$ and using the inequality (2.1), we have that
$\delta(F(t,x_n(t)),Fw)\le ad(f(t,x_n(t)),fw)+\varphi\left(2\max\left\{\delta\left(F(t,x_n(t)),f(t,x_n(t)),\delta(Fw,fw)\right)\right\}\right),$
Which implies, as $n\to\infty$, that
$\delta(u(t),Fw)\le\varphi(2\delta(Fw,u(t)).$
Thus $F\omega=\{u(t)\}$ and since (F,f) are compatible we have that $\{u(t)\}=F(t,u(t))=Ff\omega=fF\omega=\{f(t,u(t))\}$. It follows from Lemma 2.3 (1) that $u(t)$ the unique common fixed point of F and f.

Now suppose that (3) holds. Then there exists a point v in K such that $fv=u(t)$ and using the inequality (2.1), we have that
$\delta\left(Fv,F(t,x_n(t))\right)\le ad\left(fv,f(t,x_n(t))\right)+\varphi(2\max\{\delta(Fv,fv),\delta(F(t,x_n(t)),f(t,x_n(t))\}),$
Which implies, as $n\to\infty$, by $\varphi\in\phi$ and Lemma 2.1 that
$\delta(Fv,u(t))\le\varphi\left(2\delta(Fv,u(t))\right),$
Thus $Fv=\{u(t)\}$ and since (F,f) are sub compatible we have that $F(t,u(t))=Ffv=fFv=\{f(t,u(t))\}$. But again using the inequality (2.1),we deduce that
$\delta\left(F(t,u(t)),F(t,x_n(t))\right)$
$\le ad\left(f(t,u(t)),f(t,x_n(t))\right)+\delta\left(2\max\left\{\delta\left(F(t,u(t)),f(t,u(t))\right),\delta\begin{pmatrix}F(t,x_n(t)),\\ f(t,x_n(t))\end{pmatrix}\right\}\right)$
Which implies, an $n\to\infty$ by Lemma 2.1 that
$d\left(f(t,u(t)),u(t)\right)\le\delta\left(F(t,u(t)),u(t)\right)\le ad\left(f(t,u(t)),u(t)\right).$
It follows that $f(t,u(t))=u(t)$ and thus $u(t)$ is the unique common fixed point of F and f by Lemma 2.2 (1).

This completes the proof of Theorem 3.1.

Corollary 3.1 Let K be a nonempty closed subset of a complete metric space (X,d), (Ω,Σ) denotes a measurable space, C is non empty subset of K, let F be a mapping of K into $B(K)$ and f a mapping of K into itself satisfying the inequality
$\delta\left(F(t,x(t)),F(t,y(t))\right)\le ad\left(f(t,x(t)),f(t,y(t))\right)+2bmax$
$\left\{\delta\left(F(t,x(t)),f(t,x(t))\right),\delta\left(F(t,y(t)),f(t,f(t))\right)\right\},$
for all $x(t),y(t)$ in K, where $0\le a<1, 0\le 2b<1$. If F and f satisfy one of the three conditions in Theorem 3.1, then F and f have a unique common fixed point $u(t)$ in K such that $F(t,u(t))=\{u(t)\}$ if f
$\inf\left\{\delta\left(F(t,x(t)),f(t,x(t))\right):x(t)\in K\right\}=0.$

Proof Let $\varphi(t)=bt$ for all $t\in[0,\infty)$. Then $\varphi\in\phi$ and hence the conclusions of Corollary 3.1 follows from Theorem 3.1.

Theorem 3.2 Let K be a nonempty closed subset of a complete convex metric space (X, d, W), (Ω, Σ) denotes a measurable space, C is non empty subset of K, let F be a mapping of K into $B(K)$ and f a mapping of K into itself satisfying the following inequality

$$\delta\left(F(t,x(t)),F(t,y(t))\right) \leq ad\left(f(t,x(t)),f(t,y(t))\right) + (1-a)\max\begin{Bmatrix}\delta\left(F(t,x(t)),f(t,x(t))\right),\\ \delta\left(F(t,y(t)),f(t,y(t))\right)\end{Bmatrix} \tag{3.2}$$

for all x, y in K, where $0 < a < 1$. If fK is a convex subset of K such that $FK \subseteq fK$, and F and f satisfy one of the three conditions in Theorem 3.1, then F and f have a unique common fixed point in K such that $F(t,u(t)) = \{u(t)\}$.

Proof: Let $x(t) = x_0(t)$ be anarbitrary point in K and choose points $x_1(t), x_2(t), x_3(t)$ in K such that
$f(t,x_1(t)) \in F(t,x(t))$, $f(t,x_2(t)) \in F(t,x_1(t))$, $f(t,x_3(t)) \in F(t,x_2(t))$.
This can be done since fK contains FK. Then for $i = 1,2,3$ we have on using the inequaliy (3.2)

$$\begin{aligned}\delta\left(F(t,x_i(t)),f(t,x_i(t)\right) &\leq \delta\left(F(t,x_i(t)),F\left(t,x_{i-1}(t)\right)\right)\\ &\leq ad\left(f(t,x_i(t)),f(t,x_{i-1}(t)\right) + (1-a)\max\begin{Bmatrix}\delta\left(F(t,x_i(t)),f(t,x_i(t))\right),\\ \delta\left(F(t,x_{i-1}(t)),f(t,x_{i-1}(t)\right)\end{Bmatrix}\\ &\leq a\delta\left(F(t,x_{i-1}(t)),f(t,x_{i-1}(t))\right) + (1-a)\max\begin{Bmatrix}\delta\left(F(t,x_i(t)),f(t,x_i(t))\right),\\ \delta\left(F(t,x_{i-1}(t)),f(t,x_{i-1}(t))\right)\end{Bmatrix}\end{aligned}$$

and so

$$\delta\left(F(t,x_i(t)),f(t,x_i(t)\right) \leq \delta\left(F(t,x_{i-1}(t)),f(t,x_{i-1}(t))\right).$$

It follows that

$$\delta\left(F(t,x_i(t)),f(t,x_i(t)\right) \leq \left(F(t,x(t)),f(t,x(t))\right) \tag{3.3}$$

for $i = 1, 2, 3$. We shall now define a point z in K by

$$z = W\left(f(t,x_2(t)),\ f(t,x_3(t)),\ \tfrac{1}{2}\right).$$

Since fK is convex, there exists ω in K such that

$$\begin{aligned} f\omega = z &= W\left(f(t,x_2(t)),f(t,x_3(t)),\tfrac{1}{2}\right)\\ &\subseteq W\left(F(t,x_1(t)),\quad F(t,x_2(t)),\quad \frac{1}{2}\right).\end{aligned}$$

By the definition of convex structure, the inequality (3.2) and (3.3) we have that

$$\begin{aligned} d(f(t,x_1(t)),\ f\omega) &\leq, \delta\left(f(t,x_1(t)),W\left(F(t,x_1(t)),F(t,x_2(t)),\tfrac{1}{2}\right)\right)\\ &\leq \tfrac{1}{2}\left[\delta\left(f(t,x_1(t)),F(t,x_i(t))\right) + \delta\left(f(t,x_1(t)),F(t,x_2(t))\right)\right]\\ &\leq \tfrac{1}{2}[\delta\left(\left(f(t,x_i(t)),F(t,x(t))\right) + \left(F(t,x(t)),F(t,x_2(t))\right)\right]\\ &\leq \frac{1}{2}\Big[\delta\left(f(t,x(t)),F(t,x(t))\right) + ad\left(f(t,x(t)),f(t,x_2(t))\right)\\ &\qquad + (1-a)\max\left(\delta\left(F(t,x(t)),f(t,x(t))\right),\delta\left(F(t,x_2(t)),f(t,x_2(t))\right)\right\}\Big]\\ &\leq \frac{1}{2}[\delta\left(F(t,x_i(t)),f(t,x_i(t))\right) + a\delta\left(f(t,x(t)),f(t,x_1(t))\right) + a\delta(f(t,x_1(t)),f(t,x_2(t)))\\ &\qquad + (1-a)\delta(F(t,x(t)),f(t,x(t))]\\ &\leq \tfrac{2+a}{2}\delta\left(F(t,x(t)),f(t,x(t))\right) \end{aligned} \tag{3.4}$$

$$d(f(t,x_2(t)),\ f\omega) = d\left(f(t,x_2(t)),\quad W\left(f(t,x_2(t)),\quad f(t,x_3(t)),\quad \frac{1}{2}\right)\right)$$

$$\le \frac{1}{2} d(fx_2, fx_3)$$

$$\le \tfrac{1}{2}\delta\left(F(t,x(t)), f(t,x(t))\right) \qquad (3.5)$$

It follows from (3.4) and (3.5) that

$$\delta(F\omega, f\omega) \le \delta\left(F\omega, W\left(F(t,x_1(t)), F(t,x_2(t)), \frac{1}{2}\right)\right)$$

$$\le \tfrac{1}{2}\left[\delta\left(F\omega, F(t,x_1(t))\right) + \delta\left(F\omega, F(t,x_2(t))\right)\right]$$

$$\le \frac{a}{2}\left[d\left(f\omega, f(t,x_1(t))\right) + d\left(f\omega, f(t,x_2(t))\right)\right] + (1-a)\max\left\{\delta(F\omega, f\omega), \delta\left(F(t,x(t)), f(t,x(t))\right)\right\}$$

$$\le \frac{a(3+a)}{4} d\left(F(t,x(t)), f(t,x(t))\right) + (1-a)\max\left\{\delta(F\omega, f\omega), \delta\left(F(t,x(t)), f(t,x(t))\right)\right\}$$

and so

$$\delta(F\omega, f\omega) \le a\delta\left(F(t,x(t)), f(t,x(t))\right),$$

Where $a = 4 - a - \frac{a^2}{4} < 1$. Therefore

$$\inf\left\{\delta\left(F(t,x(t)), f(t,x(t))\right): x \in K\right\} \le \inf\left\{\delta(F\omega, f\omega): f\omega = W\begin{pmatrix} f(t,x_2(t)), \\ f(t,x_3(t)), \frac{1}{2}\end{pmatrix}\right\}$$

$$\le ainf\left\{\delta\left(F(t,x(t)), f(t,x(t))\right): x \in K\right\},$$

And so

$$\inf\left\{\delta\left(F(t,x(t)), f(t,x(t))\right): x \in K\right\} = 0,$$

Hence proved.

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